Digitale Signalverarbeitung WS 2025/26 – 1. Aufgabe

Analog Signals and Systems

Gruppennummer 211
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1. Exercise

$$c_1 = -5 + 3j = \sqrt{(-5)^2 + 3^2}\, e^{j\arctan\left(\frac{3}{-5}\right)} = \sqrt{34}\, e^{-j\,0.54}$$

$$c_2 = \frac{\sqrt{2}}{2} e^{-j\frac{3\pi}{4}} = \frac{\sqrt{2}}{2}\cos\left(-\frac{3\pi}{4}\right) + j\frac{\sqrt{2}}{2}\sin\left(-\frac{3\pi}{4}\right) = -\frac{1}{2} - \frac{1}{2}j$$

$$c_3 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}\, e^{j\arctan\left(\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}\right)} = e^{j\frac{\pi}{4}}$$

$$c_4 = c_1 + c_2 = (-5 + 3j) + \left(-\frac{1}{2} - \frac{1}{2}j\right) = -\frac{11}{2} + \frac{5}{2}j$$

$$c_5 = c_1 \cdot c_2 = \left(\sqrt{34}\, e^{-j\,0.54}\right)\left(\frac{\sqrt{2}}{2} e^{-j\frac{3\pi}{4}}\right) \approx \sqrt{17} e^{-j\,2.8961}$$

$$c_6 = |c_3|^2 = \left(\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}\right)^2 = 1$$

$$c_7 = \arg(c_3) = \frac{\pi}{4}$$

$$c_8 = \frac{c_1}{c_2} = \frac{-5+3j}{-\frac{1}{2}-\frac{1}{2}j} \cdot \frac{-\frac{1}{2}+\frac{1}{2}j}{-\frac{1}{2}+\frac{1}{2}j}$$
$$= \frac{(-5)(-\frac{1}{2}) + 3(-\frac{1}{2}) + j\left(3(-\frac{1}{2}) - (-5)(-\frac{1}{2})\right)}{\left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2}$$
$$= 2 - 8j$$

$$c_9 = c_1 \cdot c_1^* = \sqrt{34}\, e^{-j\,0.54} \cdot \sqrt{34}\, e^{j\,0.54} = 34$$

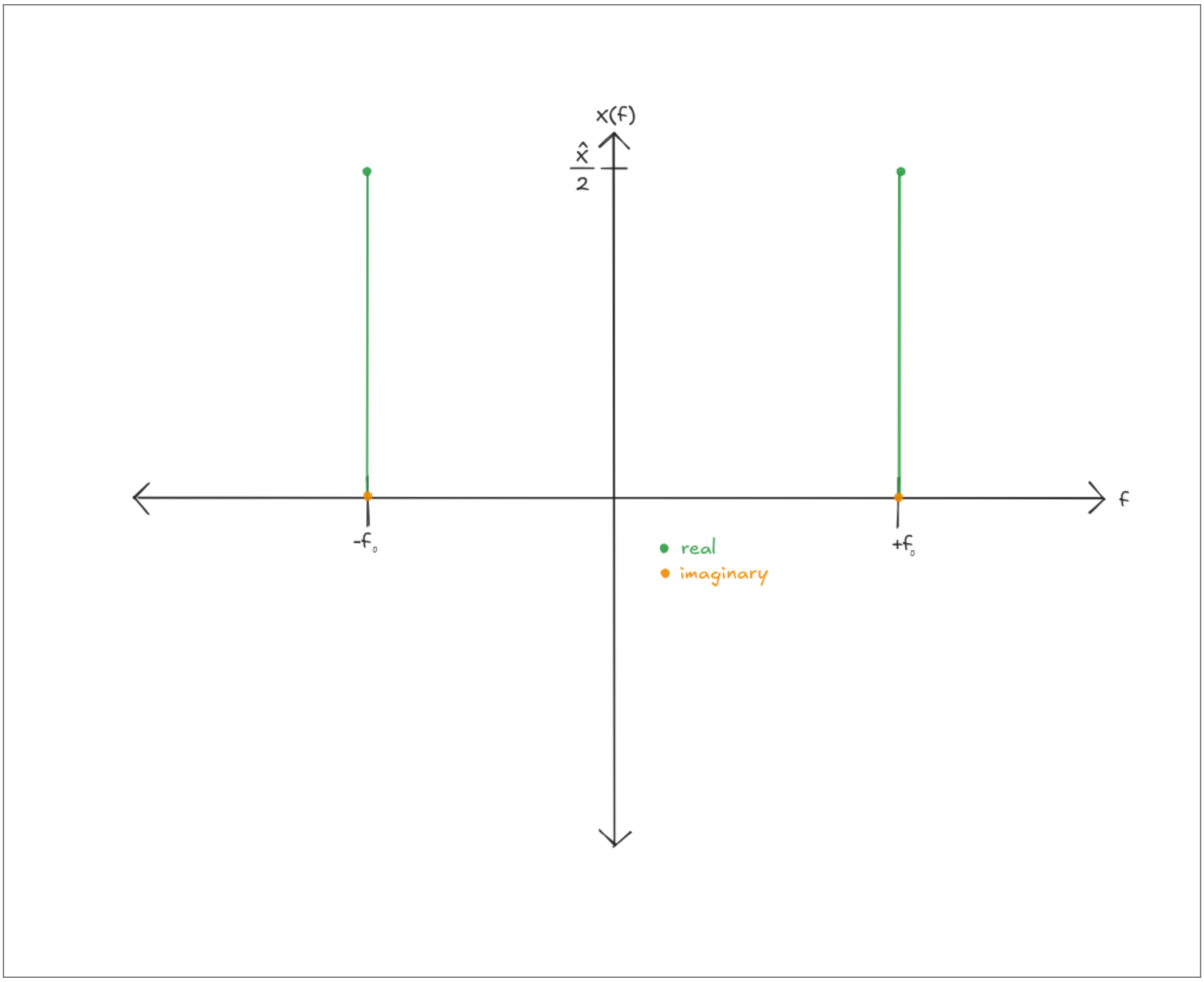
2. Exercise

$$x(t) = \hat{X}\cos(2\pi f_0 t) \iff X(f) = \frac{\hat{X}}{2}\delta(f-f_0) + \frac{\hat{X}}{2}\delta(f+f_0)$$

$$x(t) = \hat{X}(\frac{1}{2} \cdot e^{j2\pi f_0 t} + \frac{1}{2} \cdot e^{-j2\pi f_0 t})$$

$$X(f) = \hat{X}(\frac{1}{2} \cdot \delta(f-f_0) + \frac{1}{2} \cdot \delta(f+f_0))$$

$$X(f) = \frac{\hat{X}}{2} \cdot \delta(f-f_0) + \frac{\hat{X}}{2} \cdot \delta(f+f_0)$$



3. Exercise

a)

$$\sin\left(2\pi f_i(t-0.1)\right) = \sin\left(2\pi f_i t + \phi_i\right)$$

$$2\pi f_i(t-0.1) = 2\pi f_i t + \phi_i$$

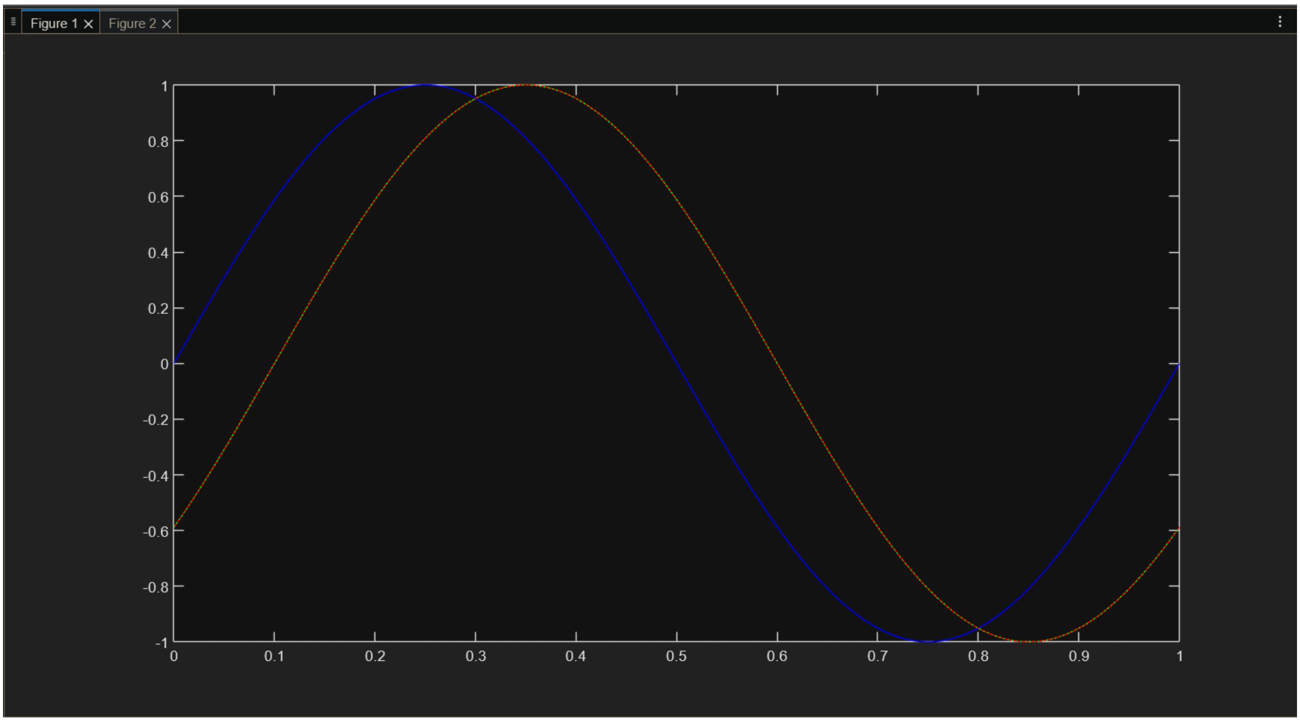
$$2\pi f_i(t-0.1) - 2\pi f_i t = \phi_i$$

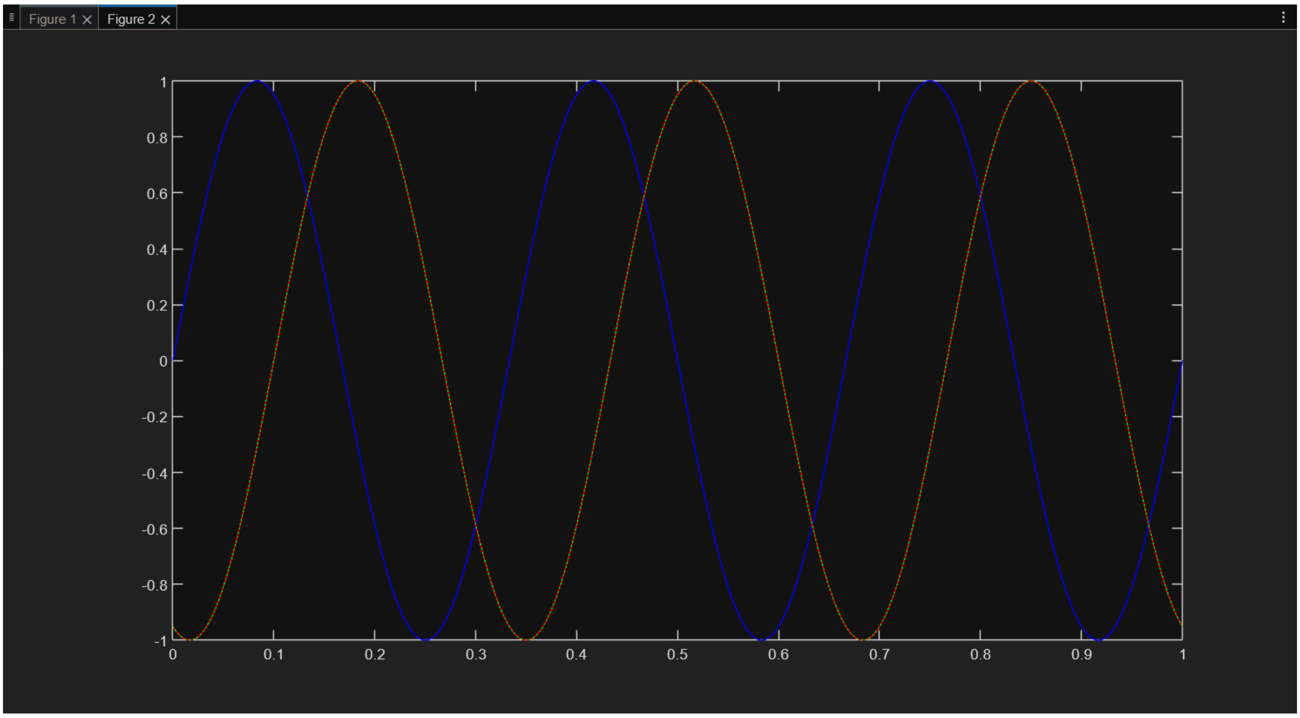
$$2\pi f_i\big((t-0.1) - t\big) = \phi_i$$

$$\phi_i = 2\pi f_i(-0.1) = -0.1 \cdot 2\pi f_i$$

The fourier transform shift theorem states $x(t-T) \quad \longleftrightarrow \quad X(f)\, e^{-j2\pi f T}$, where the angle is $2\pi f T$ and $T = 0.1$ in this case. Combining everything, we get $-0.1 \cdot 2\pi f_i$ which is equivalent to the result we got.

b) Figures:





4. Exercise

- $y(t) = (x(t))^2$

$$y(t) = (x(t))^2$$

$$y_1(t) = (\alpha x_1(t))^2 + (\beta x_2(t))^2$$

$$= \alpha^2 (x_1(t))^2 + \beta^2 (x_2(t))^2$$

$$y_2(t) = \alpha (x_1(t))^2 + \beta (x_2(t))^2$$

$$y_1(t) \neq y_2(t)$$

$$\implies \text{Not linear}$$

$$y_1(t) = (x(t-\Delta))^2$$

$$y_2(t) = y_1(t-\Delta) = (x(t-\Delta))^2$$

$$y_1 = y_2$$

$$\implies \text{Time invariant}$$

- $y(t) = x(t)sin(\Omega_0 t)$

$$y(t) = x(t)\sin(\Omega_0 t)$$

$$y_1(t) = (\alpha\, x(t)\sin(\Omega_0 t)) + (\beta\, x(t)\sin(\Omega_0 t))$$

$$y_t(t) = \alpha\,(x(t)\sin(\Omega_0 t)) + \beta\,(x(t)\sin(\Omega_0 t))$$

$$y_1 = y_2$$

$$\implies \text{linear}$$

$$y_1(t) = x(t-\Delta)\sin(\Omega_0 t - \Delta)$$

$$y_2(t) = y_1(t-\Delta) = x(t-\Delta)\sin(\Omega_0 t - \Delta)$$

$$y_1 = y_2$$

$$\implies \text{Time invariant}$$

5. Exercise

a) System A is linear because there is no difference between the plot $T[x_1(t)+x_2(t)]$ and $T[x_1(t)] + T[x_2(t)]$. You can also see in the error plot that the error is small. It is not exactly zero becuase rounding errors.

b) Input and output are not proportional in a non linear system and therefore the error change is greater than the one of the linear system

c) The output sound of System B does differ when comparing $T[x_1(t) + x_2(t)]$ and $T[x_1(t)] + T[x_2(t)]$. But is the same when using System A. This matches the differences in the graphs.