

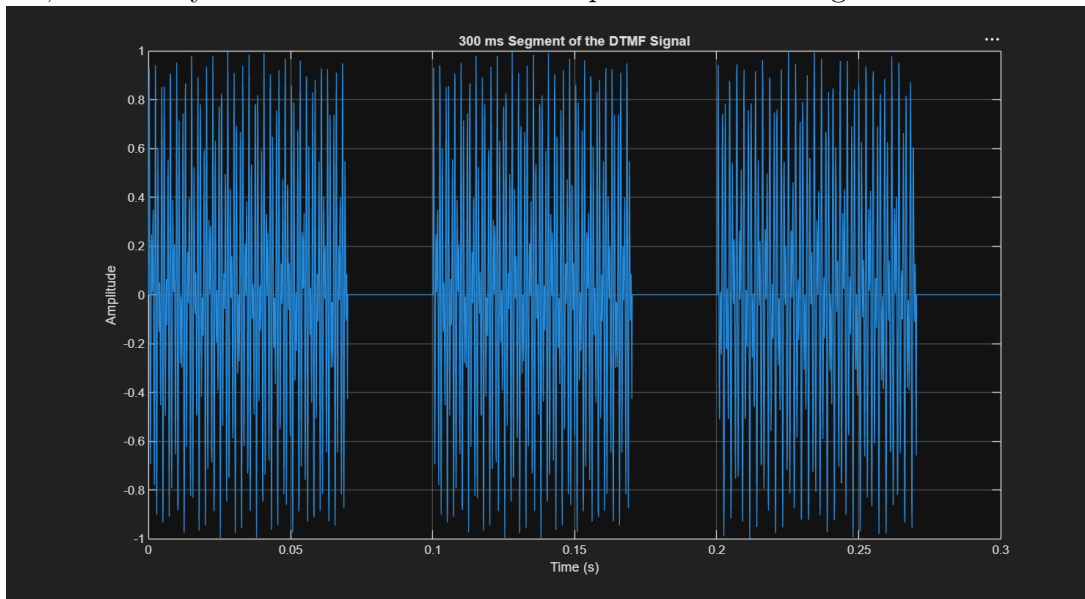
Digitale Signalverarbeitung SS 2025/26 – 5. Aufgabe

STFT, z-Transform, Digital Filters

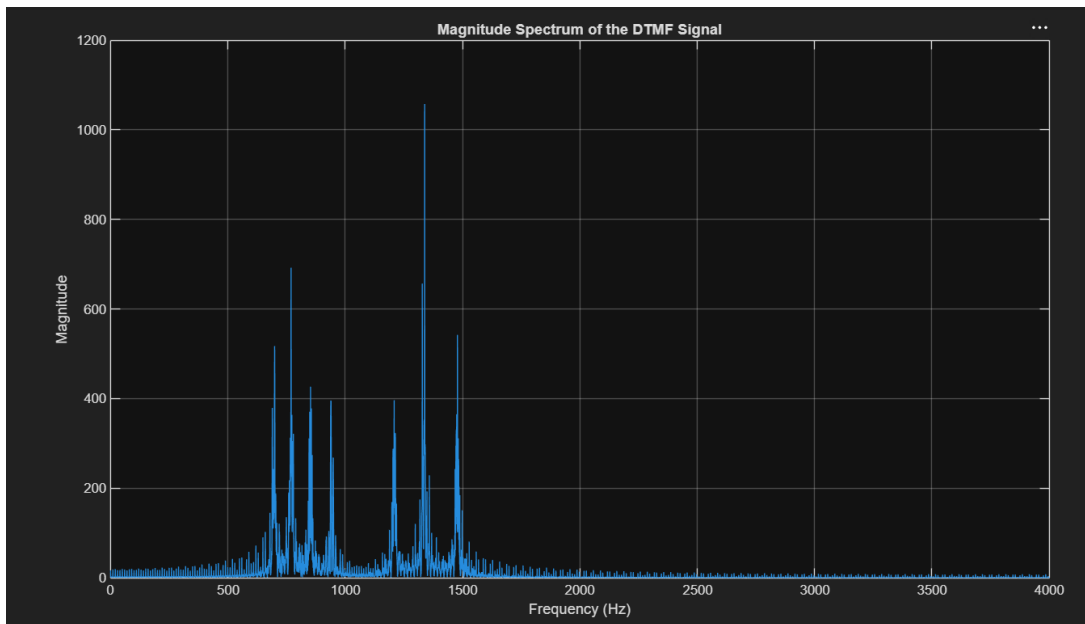
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1. Exercise *FFT in Audio Signal Processing – Short Time Fourier Transform*

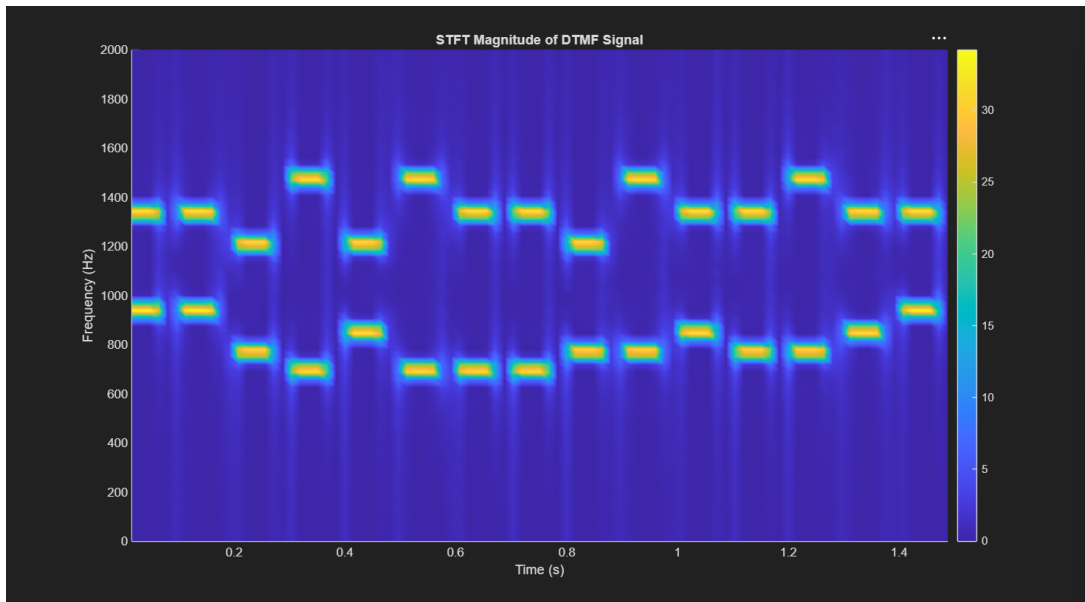
a) No, because you can't make out the frequencies in the signals without a Fourier transform



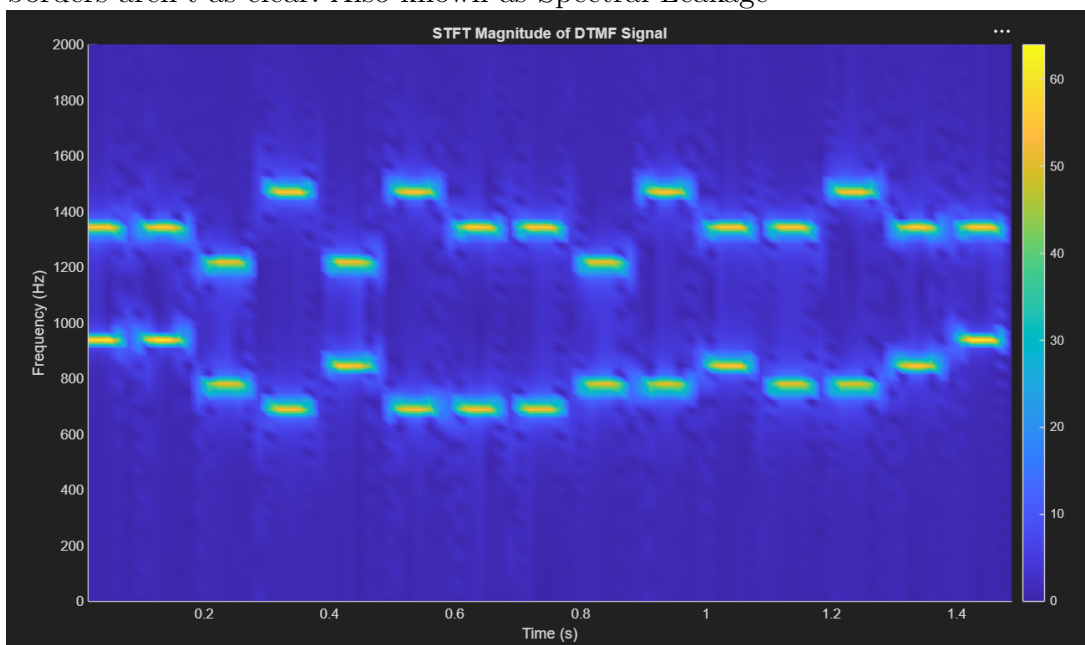
b) :



c) :

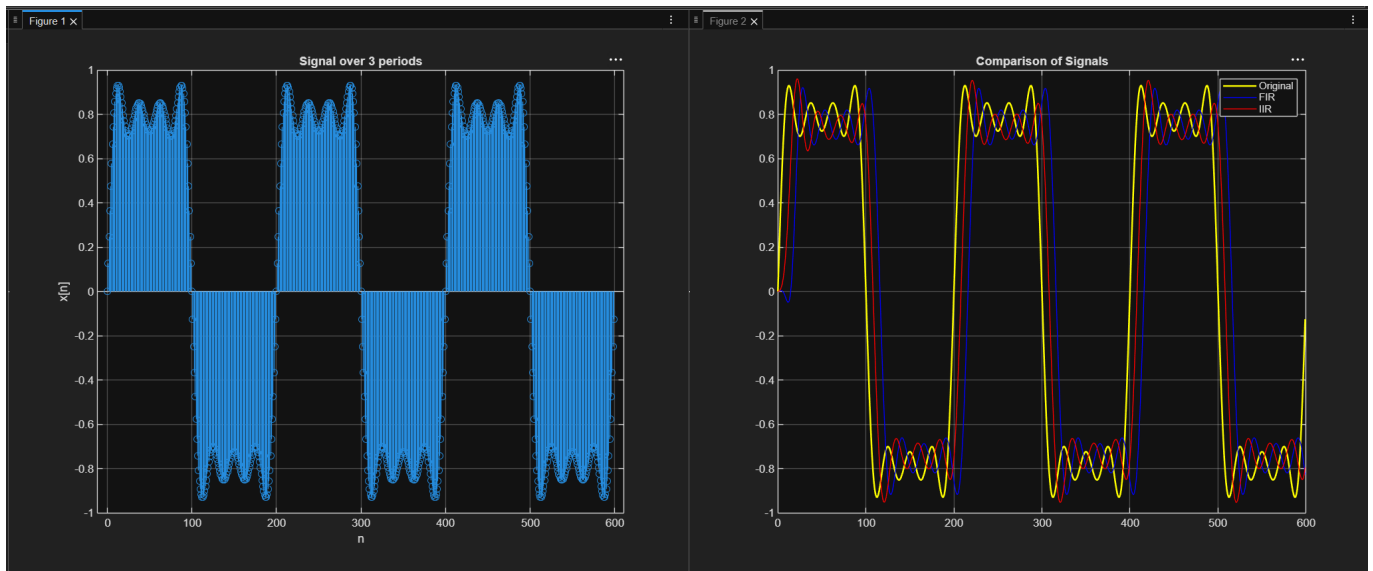


- d) The actual frequencies are more readable with the hamming windows. Without them, the borders aren't as clear. Also known as Spectral Leakage



- e) Decoding it with the figure in (c) gives us the following number sequence: 004373224685680
- f) The diagram in (b) contains all frequencies of the entire signal, and we can not determine which number sequence it is. The signal in (c) looks at the fourier transform at different locations along the signal to localize what frequencies are present at certain times, and we can now easily see the encoding. An example for this is the landline phone system where pressing numbers will play different combinations of tones into the phone line, letting the system on the other side determine which number you have dialed.

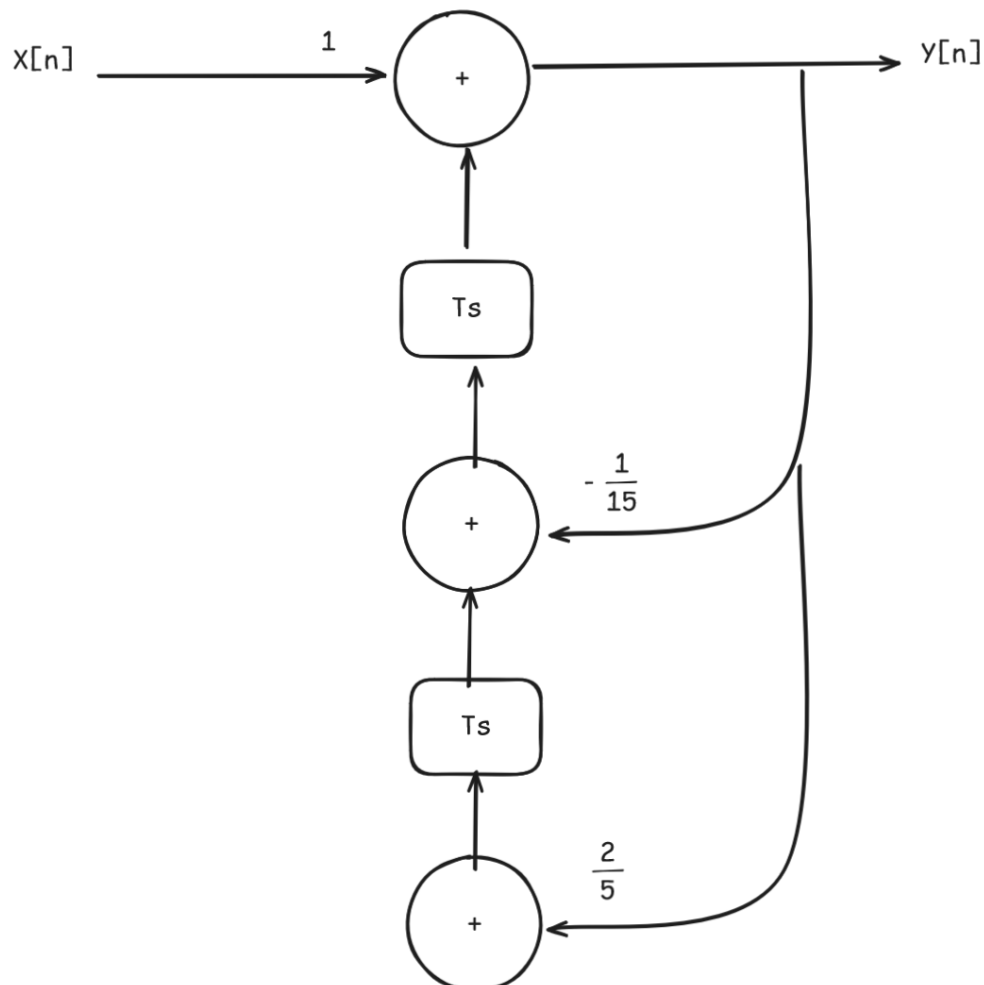
2. Exercise *Signal Distortion and Group Delay*



- **FIR Filter:** The FIR filter kept its shape but is a little bit delayed on the phase.
- **IIR Filter:** The IIR filter lost some of the shape similarity to the original signal and is also a little delayed, but uses fewer coefficients, which means that it also takes less memory and time to compute

3. Exercise z -Transform

a) :



b) IIR, because its a recursive LTI System.

c)

$$y[n] = x[n] - \frac{1}{15}y[n-1] + \frac{2}{5}y[n-2]$$

$$Y(z) = X(z) - \frac{1}{15}z^{-1}Y(z) + \frac{2}{5}z^{-2}Y(z)$$

$$Y(z) + \frac{1}{15}z^{-1}Y(z) - \frac{2}{5}z^{-2}Y(z) = X(z)$$

$$Y(z) \left(1 + \frac{1}{15}z^{-1} - \frac{2}{5}z^{-2} \right) = X(z)$$

$$Y(z) = \frac{X(z)}{1 + \frac{1}{15}z^{-1} - \frac{2}{5}z^{-2}}$$

$$Y(z) = \frac{X(z)}{1 + \frac{1}{15}z^{-1} - \frac{2}{5}z^{-2}} \cdot \frac{z^2}{z^2}$$

$$Y(z) = \frac{X(z)z^2}{z^2 + \frac{1}{15}z - \frac{2}{5}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^2}{z^2 + \frac{1}{15}z - \frac{2}{5}}$$

d) Poles:

$$0 = z^2 + \frac{1}{15}z - \frac{2}{5}$$

$$z_1 = 0.6$$

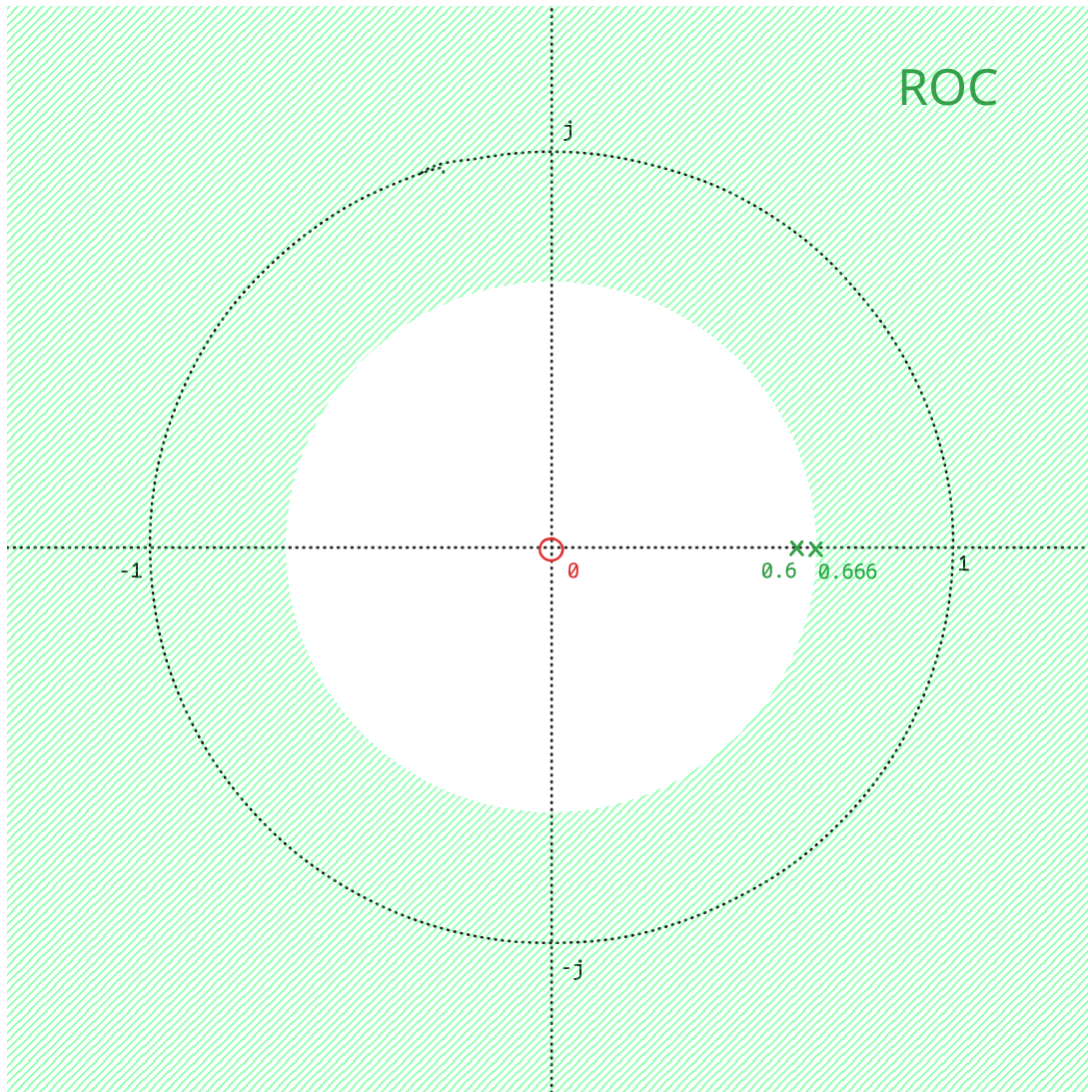
$$z_2 = -0.6\bar{6}$$

Zeros:

$$0 = z^2$$

$$z_{1,2} = 0$$

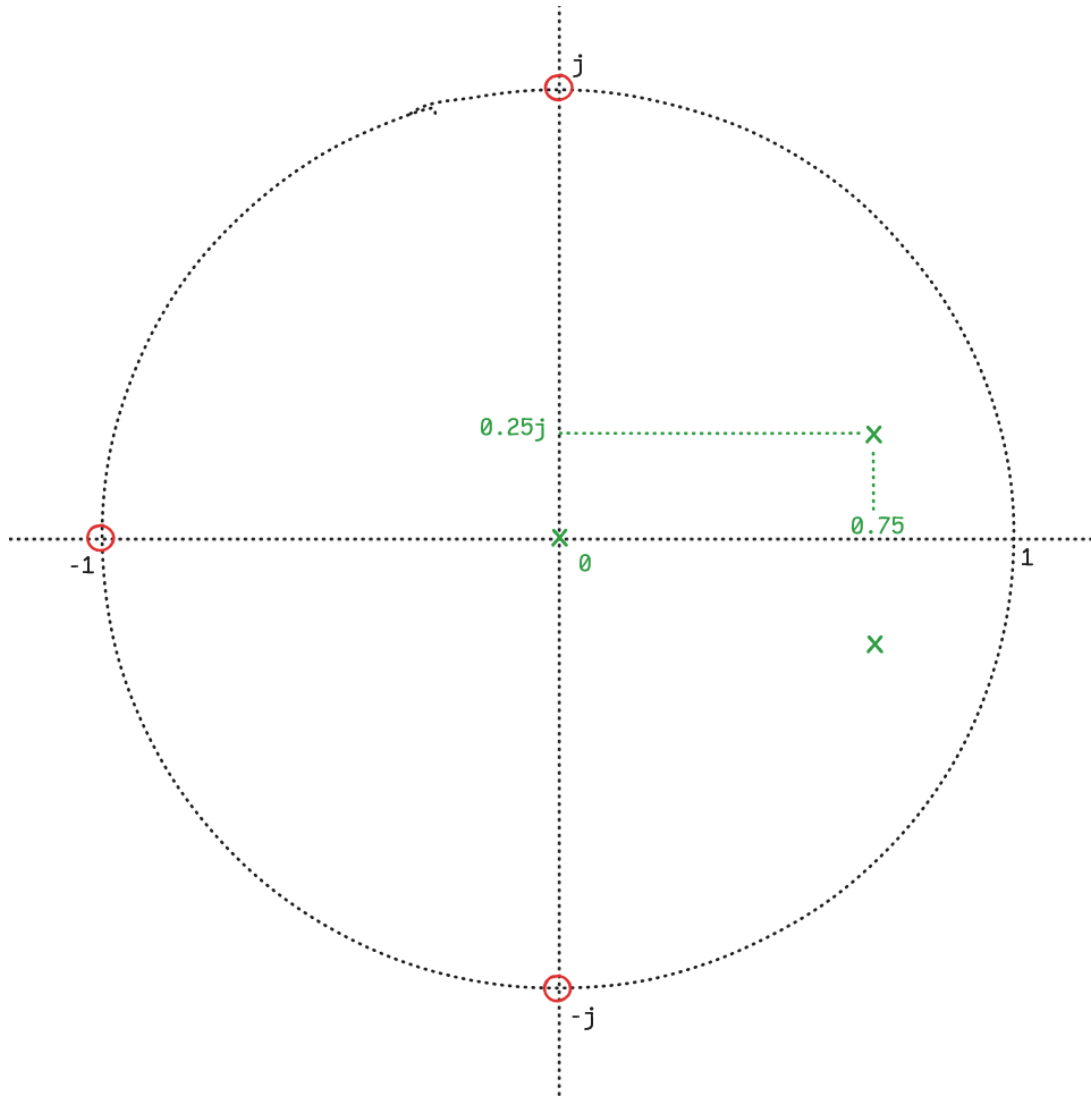
Plot:



e) The system is BIBO stable because $|p_i| < 1$ holds for all poles.

4. Exercise *Recursive Filter*

- a) Yes the coefficients are real valued, because all non real poles and zeros appear in complex conjugate pairs
- b) :



c)

$$H(z) = b_0 z^{M-N} \cdot \frac{(z+1)(z-j)(z+j)}{(z-0)(z-(0.75+0.25j))(z-(0.75-0.25j))}$$

$$H(z) = b_0 z^{3-3} \cdot \frac{(z+1)(z-j)(z+j)}{(z-0)(z-(0.75+0.25j))(z-(0.75-0.25j))}$$

$$H(z) = b_0 z^0 \cdot \frac{(z+1)(z-j)(z+j)}{(z-0)(z-(0.75+0.25j))(z-(0.75-0.25j))}$$

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$$H(z) = b_0 \cdot \frac{(z+1)(z-j)(z+j)}{(z-0)(z-(0.75+0.25j))(z-(0.75-0.25j))}$$

$$H(z) = b_0 \cdot \frac{z^3 + z^2 + z + 1}{z^3 - 1.5z^2 + 0.625z}$$

This was for the z^{+i} case. Now follows the z^{-i} case.

$$H(z) = b_0 \cdot \frac{z^3 + z^2 + z + 1}{z^3 - 1.5z^2 + 0.625z}$$

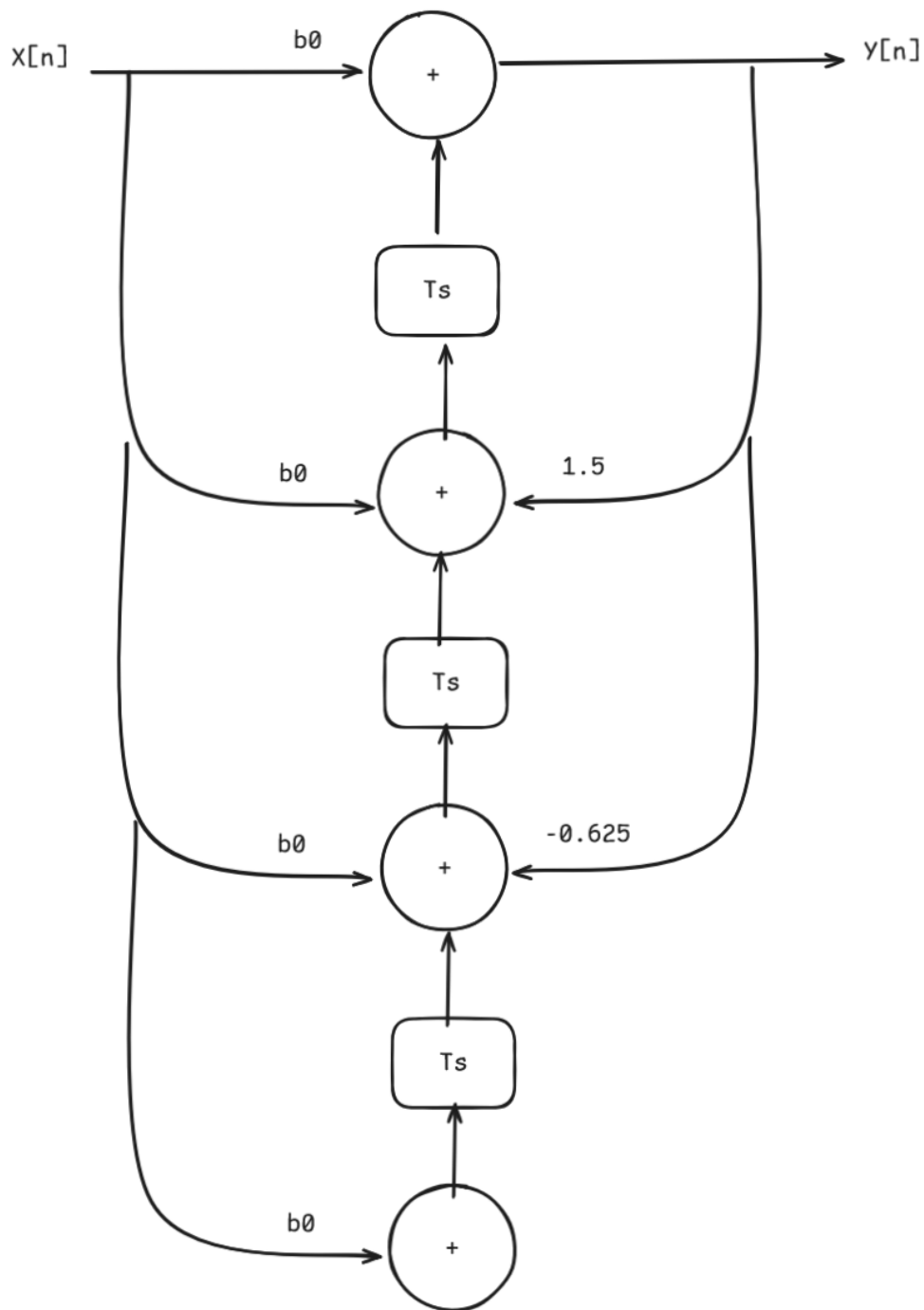
$$H(z) = b_0 \cdot \frac{z^3 + z^2 + z + 1}{z^3 - 1.5z^2 + 0.625z} \cdot \frac{z^{-3}}{z^{-3}}$$

$$H(z) = b_0 \cdot \frac{1 + z^{-1} + z^{-2} + z^{-3}}{1 - 1.5z^{-1} + 0.625z^{-2}}$$

d) With the Direct Form I and the $H(z)$ from above, we receive the following LTI System:

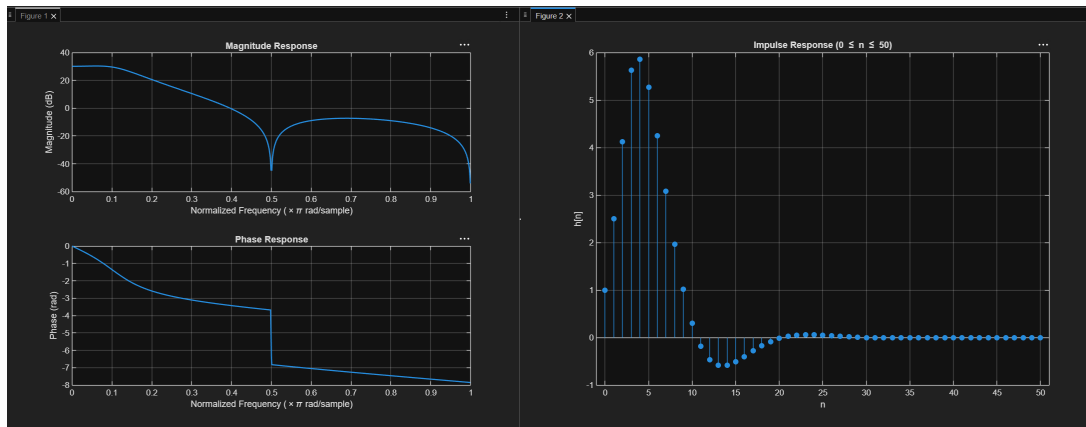
$$y[n] = b_0x[n] + b_0x[n-1] + b_0x[n-2] + b_0x[n-3] + 1.5y[n-1] - 0.625y[n-2]$$

See the following block diagram:



e) (see f)

f) :



5. Exercise *Lowpass Filter Design*