Digitale Signalverarbeitung SS 2025/26 – 4. Aufgabe

Reconstruction, DFT, FFT

Gruppennummer 211

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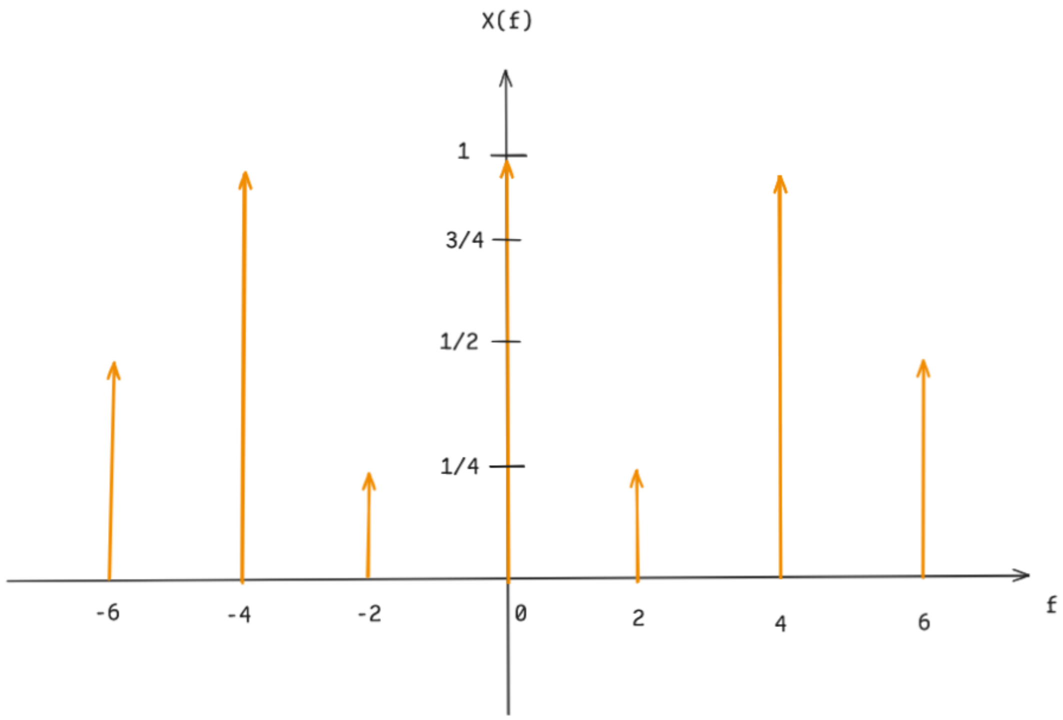
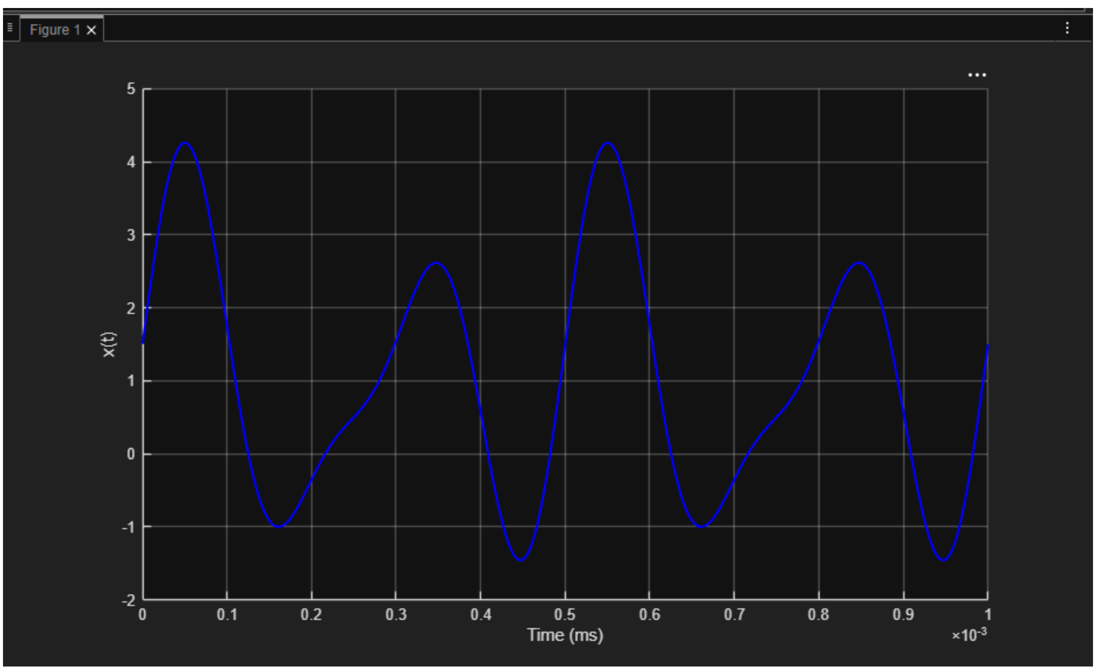
1. Exercise *Reconstruction*

a) :





b) :

$$x(t) = 1 + 0.5\cos(2\pi f_1 t) + 2\sin(2\pi f_2 t) + \sin(2\pi f_3 t)$$

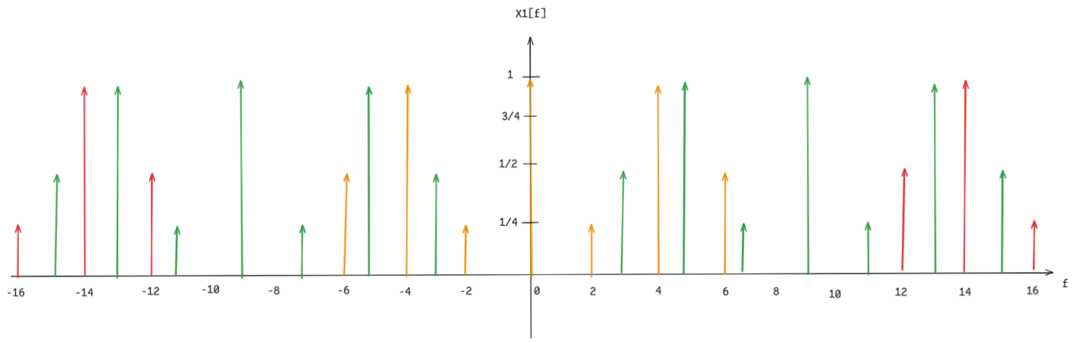
Calculation of $x_1[n]$:

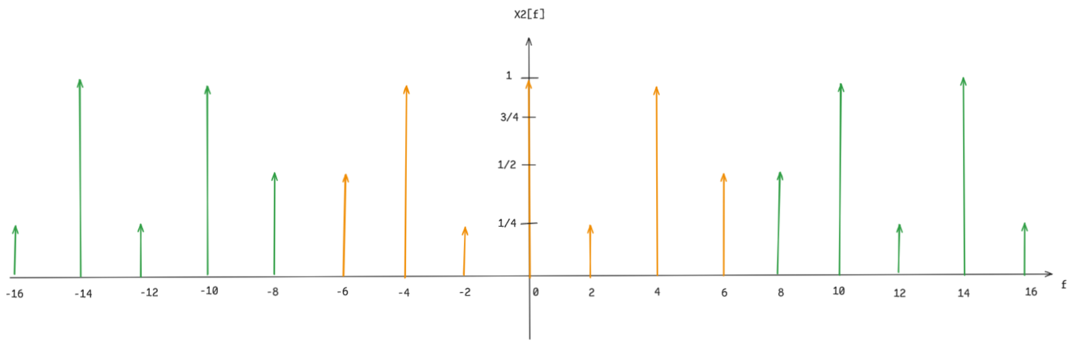
$$\begin{aligned}
x(t) &= 1 + 0.5\cos(2\pi f_1 t) + 2\sin(2\pi f_2 t) + \sin(2\pi f_3 t) \\
&= 1 + 0.5\cos(2\pi 2000t) + 2\sin(2\pi 4000t) + \sin(2\pi 6000t) \\
x_1[n] &= 1 + 0.5\cos(2\pi \frac{2000}{9000} n) + 2\sin(2\pi \frac{4000}{9000} n) + \sin(2\pi \frac{6000}{9000} n) \\
&= 1 + 0.5\cos(\frac{4\pi}{9} n) + 2\sin(\frac{8\pi}{9} n) + \sin(\frac{4\pi}{3} n)
\end{aligned}$$

$$\sin(\frac{4\pi}{3}) = -\sin(\frac{2\pi}{3})$$
$$\implies x_1[n] = 1 + 0.5\cos(\frac{4\pi}{9} n) + 2\sin(\frac{8\pi}{9} n) - \sin(\frac{2\pi}{3} n)$$

Calculation of $x_2[n]$:

$$\begin{aligned}
x_1[n] &= 1 + 0.5\cos(2\pi \frac{2000}{14000} n) + 2\sin(2\pi \frac{4000}{14000} n) + \sin(2\pi \frac{6000}{14000} n) \\
&= 1 + 0.5\cos(\frac{2\pi}{7} n) + 2\sin(\frac{4\pi}{7} n) + \sin(\frac{6\pi}{7} n)
\end{aligned}$$





c) :

$$f = \frac{\Omega f_s}{2\pi}$$

Calculations for $x_1(t)$:

$$f_1 = \frac{\Omega_1 f_s}{2\pi} = \frac{\left(\frac{4\pi}{9}\right) 9000}{2\pi} = 2000$$

$$f_2 = ... = 4000$$

$$f_3 = \frac{\Omega_3 f_s}{2\pi} = \frac{\left(-\frac{2\pi}{3}\right) 9000}{2\pi} = -3000$$

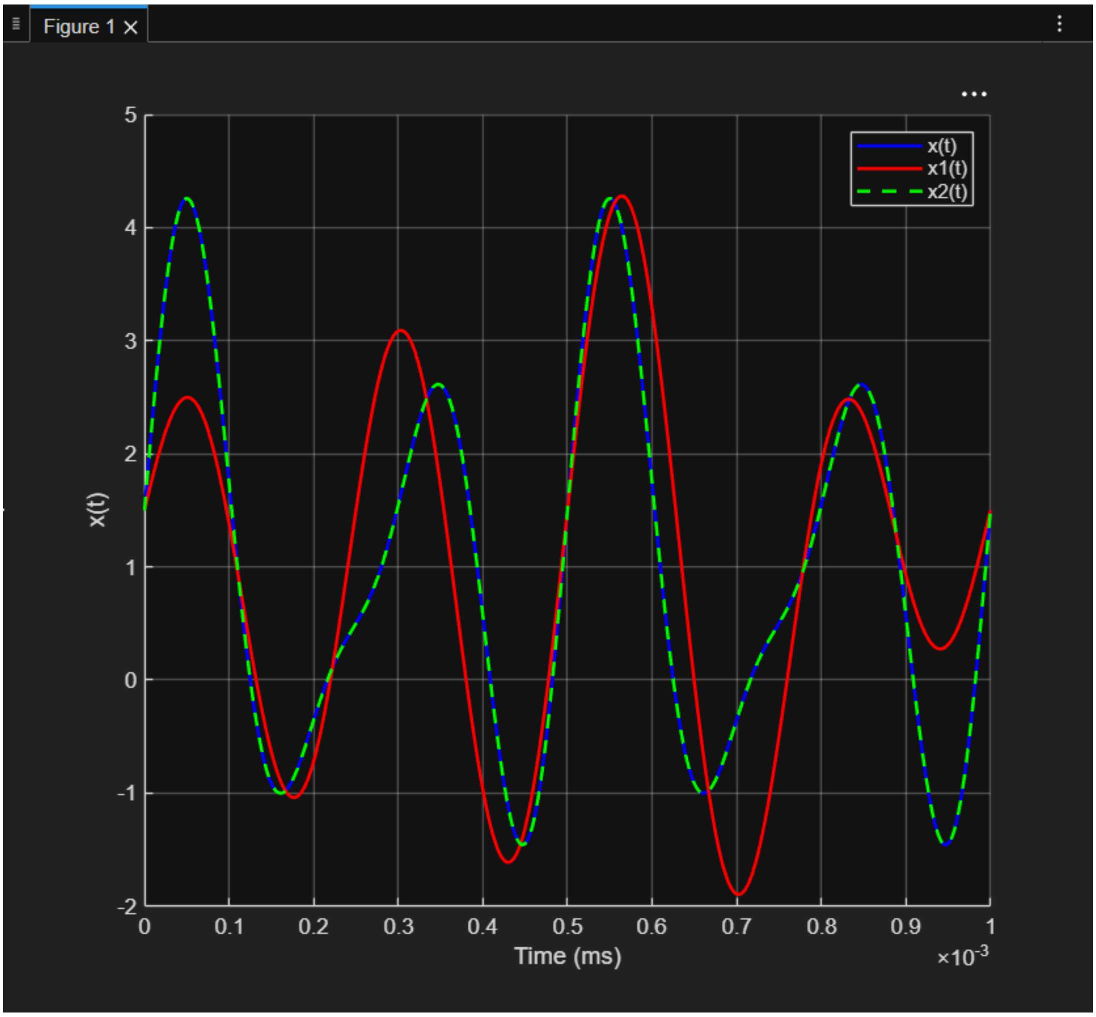
Calculations for $x_2(t)$:

$$f_1 = ... = 2000$$

$$f_2 = ... = 4000$$

$$f_3 = ... = 6000$$

Where '...' is just the formular at the top injected with the Ω values of b)



2. Exercise *DFT Theory*

a)

$$f_s = \frac{1}{T_s} = \frac{1}{0.001} = 1000\text{Hz}$$
$$\Delta f = \frac{f_s}{N} = \frac{1000}{100} = 10\text{Hz}$$

b)

$$\text{In terms of samples} : 100$$
$$\text{In terms of frequency} : N \cdot \Delta f_s = 1000\text{Hz}$$
$$\text{In terms of angular frequency} : \omega = 2\pi\frac{f}{f_s} = 2\pi\frac{1000}{1000} = 2\pi$$

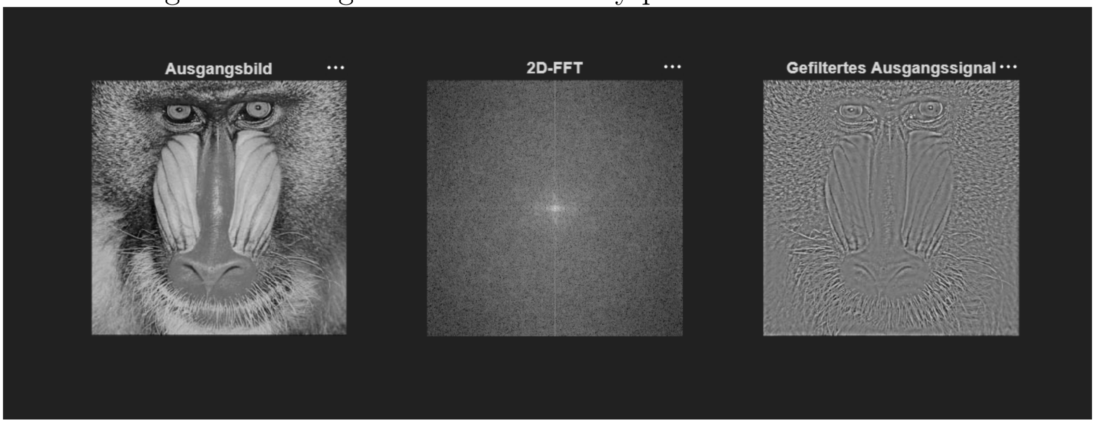
c) Computers can calculate it more efficently when it is working with powers of two

d) $\Delta f = \frac{f_s}{N} = \frac{1000}{128} = 7.8125\text{Hz}$

e) Even though we have not gained new information, we have a finer resolution of the frequency axis. This can make it easier to get closer to the actual value.

3. Exercise *FFT in Image Processing*

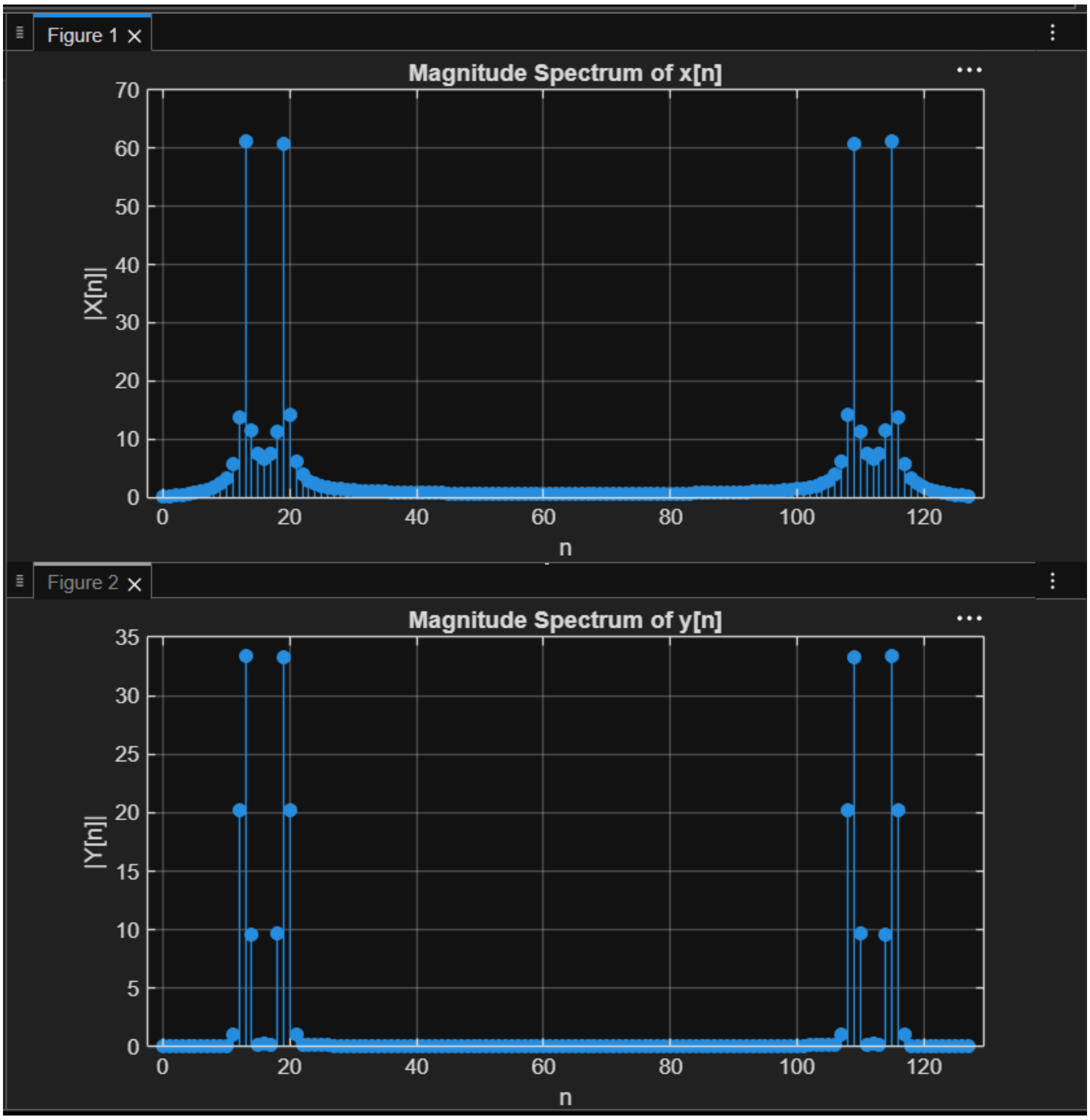
Approach: We want to remove the low frequencies (high pass filter) to keep the areas of the image where the lightness changes relative to nearby pixels



4. Exercise *Window Effects of the DFT*

a) (see b))

b) (see assignment4_4.m)



c) Rectangular window: Clear peaks but also many neighbors have fairly high values Hamming window: Still visible peaks, that may be a bit wider, but now clearly separated from neighbors and more visually readable.

d) We can choose freqencies that align with our 128 steps, for example f1 = 10/N and f2 = 15/N. With this, the signal fits exactly within our window, each cosine has a integer number of cycles it completes

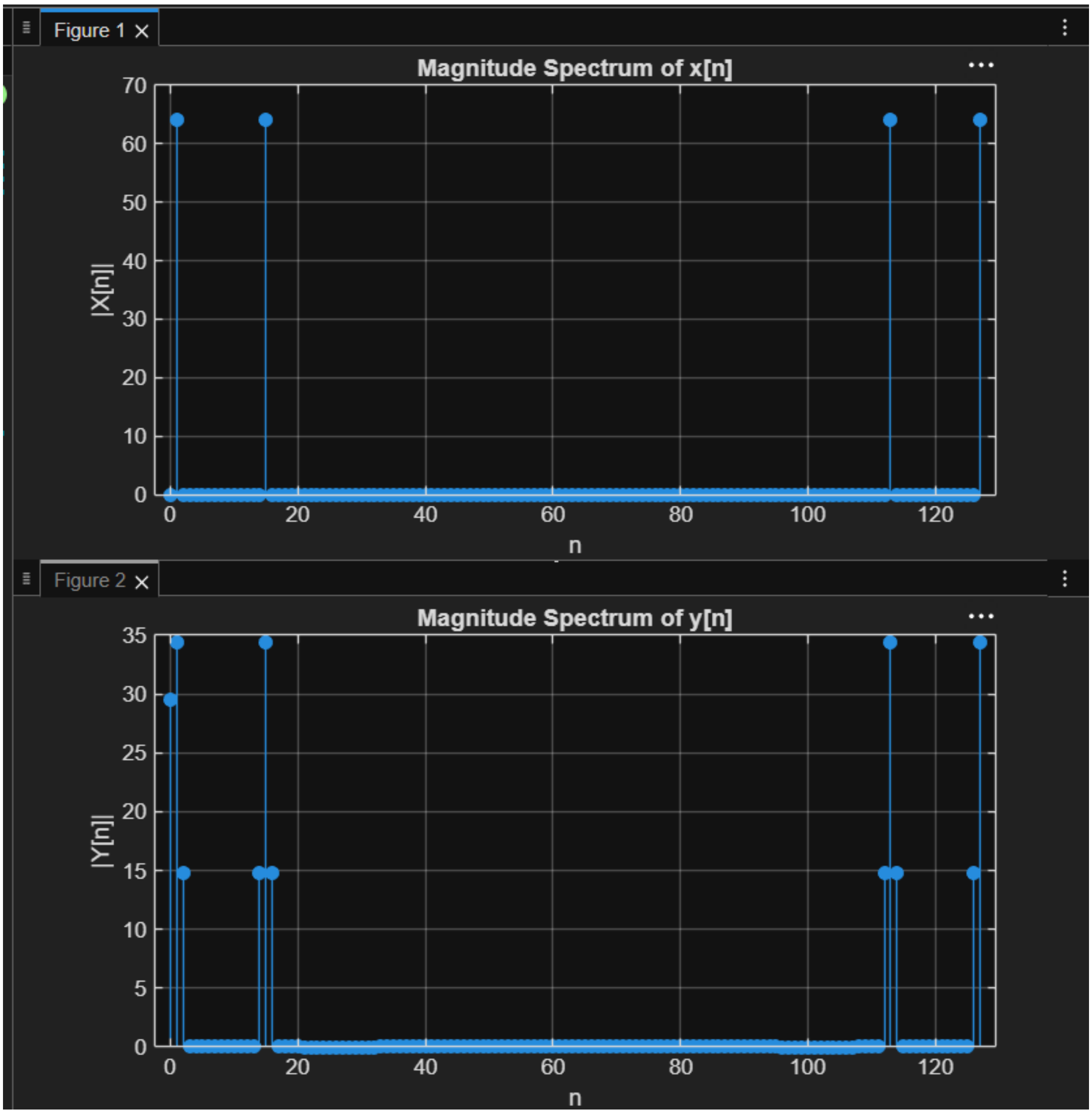


Figure 1
Magnitude Spectrum of x[n]
|X[n]|
n
0
10
20
30
40
50
60
70
0
20
40
60
80
100
120
Figure 2
Magnitude Spectrum of y[n]
|Y[n]|
n
0
5
10
15
20
25
30
35
0
20
40
60
80
100
120